On the Stability Region of Two-User Slotted ALOHA with Cooperative Relays

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Abstract—In this work, we analyze the stability region of the slotted ALOHA random access network with cooperative users. With cooperation, the packets corresponding to each user can be transmitted to the destination through the multiple relaying paths formed by the other users. The diversity introduced by the multi-path relaying reduces significantly the probability of outage. We propose two queuing strategies for the cooperative system. In the first strategy, we assume that each user is equipped with two buffers: a source buffer, to record its own packets, and the relay buffer, to store the packets received from its partner. When both buffers are non-empty, the user chooses randomly among the two buffers based on a predetermined probability. An inner bound of the stability region is derived and is shown to outperform the non-cooperative system. In the second case, we assume that each user utilizes only one buffer to store all packets. The relaying packets are stored at the front of the queue at the cooperative user upon arrival. The second scheme outperforms the non-cooperative case only for certain channel conditions and arrival rates. However, the scheme has low complexity and can be implemented with little changes to the original system.

I. INTRODUCTION

Cooperative communications, as proposed in [1]–[3], refers to a wireless system where the users cooperate in relaying each others messages to the destination. With cooperation, each user can choose among multiple relaying paths to transmit its messages, and an outage occurs only when the quality of all channel paths fall below the desired level simultaneously. The advantages are provided through the spatial diversity inherent in multi-user systems. Many coding, modulation and cooperative signal processing techniques have been proposed in the literature based on different relaying methods, such as decode-and-forward, amplify-and-forward [2] and coded cooperation [3] etc, to maximize the advantages of cooperation. These works focus on improving the physical-layer performances such as diversity gains, capacity and probability of error. The goal of this work is to study the advantages of physical-layer cooperation in the medium access control (MAC) layer.

Let us consider a slotted ALOHA random access protocol [4], [5] where each user transmits with independent probabilities in each time slot. The system has been studied extensively for nearly three decades, but continues to receive considerable interest due to advances in physical-layer technologies, such as multiple-packet reception (MPR) [6] or channel-aware transmission controls [6], [7] etc. However, even with multi-user detection or channel-aware transmissions, the throughput of each user is still limited by their respective channel qualities. In this case, users that often experience a bad channel may accumulate backlogged packets more easily than those with good channels. Furthermore, if the user experiences a good channel and rapidly clears out the messages in its queue, the good channel will be left idle while the user with a bad channel remains backlogged. The inefficient use of channel resources can be resolved with the use of cooperative relaying.

The main contribution of this work is to apply the physical-layer cooperation in the slotted ALOHA system and to show how cooperation can improve the MAC performances. Two queuing strategies are proposed with respect to the cases: (a) with relay buffers; and (b) without relay buffers. In the first case, each user is equipped with both the source buffer, to record its own packets, and the relay buffer, to store the packets received from its partner. When both buffers are non-empty, the user chooses randomly among the two buffers based on a predetermined probability, which depends on the priority given to each buffer. An inner bound of the stability region is derived and is used to illustrate the improvements over the non-cooperative system. In the second case, we assume that each user utilizes only one buffer to store both the source and the relay packets. The relay packets are stored at the front of the queue upon arrival and are dropped from the source user’s queue to avoid having the two users collide while transmitting the same packet. The second scheme outperforms the non-cooperative case only for certain channel conditions and arrival rates. Nonetheless, the scheme has low complexity and can be implemented with little changes to the original system.

To quantify the advantages, we analyze the stability region of the two-user cooperative slotted ALOHA system. The stability region [8] is the set of arrival rates for which there exists a transmission policy that allows all queues to remain finite with probability one. It has been studied extensively in the past for the cases with no cooperation and perfect transmission channels, e.g. [8], [9]. Even in these cases, the exact stability region is known only for a system with 2 users [8] while inner and outer bounds are derived for the general case. The difficulty lies in the complex interaction between the queues located at different users. In this work, we analyze the stability region of the cooperative slotted ALOHA system for the two-user case. The inner and outer bounds for the general case will be the focus of our future investigation. Recently, the
stability issues have also been studied for the network with cognitive relays under the TDMA setting [10]. Our work is different in the sense that random access is considered instead of TDMA scheduling and cooperation is considered instead of pure relaying.

II. SYSTEM MODEL AND QUEUING STRATEGIES

Consider a slotted ALOHA system with two users transmitting to a base station through a shared channel as shown in Fig. 1. The users transmit in synchronized time slots with intervals equal to the transmission time of a packet. The length of each time slot is set to 1, w.l.o.g., and the transmission in the m-th time slot takes place during the time \( t \in [m, m + 1) \).

For \( i \in \{1, 2\} \), let \( \lambda_i \) packets/slot be the arrival rate for the packets generated by user \( i \) and let \( B_i \) be the buffer, with infinite capacity, that is used to record these packets.

Let \( Q_i[m] \) be the number of packets queued in the buffer \( B_i \) at the beginning of the m-th slot. If \( B_i \) is non-empty at the beginning of a time slot, user \( i \) will make a transmission attempt in the current time slot with probability \( p_i \). The probability is independent for each user and over each time slot. The transmission attempt of a user will be successful only if no other users are transmitting in the same time slot, which is the well-known collision model [5]. However, even if a collision does not occur, the packet still may not be received correctly due to bad channel conditions. Let \( \psi_i \) be the probability that a packet from user \( i \) is received correctly at the base station given that no collision occurred. This is used to model the quality of the user’s channel. Assume that the probability is also independent for each user and over each time slot. At the end of each time slot, the base station will feedback a \((0, 1, e, c)\) message to the users, with 0 indicating that the slot is idle, 1 indicating a success, \( e \) indicating a failure due to the bad channel condition and \( c \) indicating a failure due to collision.

In cooperative systems, each user is allowed to help the other users relay their messages to the base station. We assume that the cooperating users are sufficiently close such that the packets transmitted over the inter-user channel are always correctly received by the other user. The user that generated the packet is called the source user while the user relaying the packet is called the cooperative user. We consider a half-duplex system where the cooperating user can receive (and store) the message from the source user only if it is not transmitting in the same time slot. In this case, cooperation may potentially reduce the bandwidth efficiency. However, due to the broadcast nature of the wireless medium, each user is able to overhear the packets transmitted by the source to the destination and will have access to the packet that failed to reach the destination without having to allocate a separate time slot for inter-user communication.

In the following sections, we study the stability region of the slotted ALOHA system for two cooperative queuing strategies: (1) the case with relay buffers and (2) the case without relay buffers. The stability region [8] of the two-user slotted ALOHA system is the closure of the set of arrival rates \( \lambda_1 \) and \( \lambda_2 \) such that the system is stable. The system is said to be stable [8] if there exists \( p_1, p_2 \) such that the Markov chain is ergodic, i.e., the length of the queues remain finite with probability 1. Throughout this paper, we shall utilize the results given by Loynes in [9], where it shows that the system is stable if the departure rate of each user is greater than their respective arrival rates; on the other hand, it is unstable if the departure rate of some user is smaller than its arrival rate.

Strategy I: With Relay Buffers

In Strategy I, we assume that each user is equipped with a relay buffer to store the messages received from the source user. Let \( RB_i \) be the relay buffer of user \( i \) and let \( RQ_i[m] \) be the number of packets stored in \( RB_i \) at the beginning of the m-th slot. If user \( i \) is to transmit in the m-th slot (which is determined by the probability \( p_i \)) and both \( B_i \) and \( RB_i \) are non-empty, user \( i \) will transmit a packet from \( B_i \) with probability \( q_i \) and from \( RB_i \) with probability \( q_i = 1 - q_i \). If one of the buffers is empty, the packet from the non-empty buffer will be chosen to transmit. When a packet is transmitted successfully, the users will receive the feedback 1 and the packet will be removed from the buffers of both the source and the cooperative users. Three choices of \( q_i \) are considered:

(i) Source priority: \( q_i = 1, \forall i \), such that the packet from the source buffer is transmitted whenever it is non-empty;
(ii) Relay priority: \( q_i = 0, \forall i \), such that the packet from the relay buffer is transmitted whenever it is non-empty;
(iii) Adaptive priority: the probabilities \( q_i, \forall i \), are chosen to maximize the stability region for different channel conditions and transmission probabilities \( p_i, \forall i \).

Strategy II: Without Relay Buffers

In Strategy II, we assume that each user uses only one buffer to store all the packets. The packets received from the source user are stored at the front of the queue and are given the highest priority to transmit. To avoid collision between identical packets, which are transmitted by two separate users, the packets are dropped from the transmitting user as long as no collision occurs, in which case, either the base station or the cooperative user will have received the packet correctly. In Strategy II, an unsuccessful transmission from the source user will contribute as an arrival to the relay user if no collision occurs, which may potentially cause the user to
become unstable. On the other hand, since the packets are dropped as long as no collision occurs, the departure rate will be as high as the case with perfect channels, i.e., $\psi_i = 1$.

In strategy I, the collision of identical packets can be controlled with $q_i$, but it cannot be completely avoided if $q_i < 1$. However, diversity is gained in this case. In strategy II, we eliminate the probability of collision between two identical packets, but there may be a loss in diversity. Also, in the latter case, the performance of a user with good channel conditions may be compromised by its partner’s low quality channel.

III. Stability Region of Cooperative Slotted ALOHA with Relay Buffers

In strategy I, the stability of the two users involves the interaction between the 4 queues $B_1, B_2, RB_1, RB_2$. This makes the problem considerably more difficult than the non-cooperative case. Hence, we were not able to completely characterize the stability region, but the inner bounds that we derive are sufficient to illustrate the advantages of cooperation.

Let us consider the case where both users are fully loaded, which is defined as the case where there is always a packet to transmit in each user’s source buffer. The boundary of the arrival rates that are stable in this case serves as an inner bound for the actual region since the collision occurs with a non-zero probability in each time slot and, thus, reduces the amount of packet arrivals that the system can sustain. In this strategy, a packet transmitted from the source buffer will be stored in the relay buffer of the cooperative user if no collision occurred and the packet failed to reach the base station. The same packet will be stored at both users and will be dropped simultaneously when one of the retransmissions is successful. Since no new packets are transmitted by the source user before the backlogged packet is received by the base station, the relay buffer does not record more than 1 packet, i.e., $RQ_1[m], RQ_2[m] \in \{0, 1\}, \forall m$. Therefore, the stability of $RB_1$ and $RB_2$ is not a concern.

Let $(RQ_1[m], RQ_2[m])$ be the state of the relay buffers at time $m$, which forms a Markov Chain with the 4 states $S_0 : (0, 0), S_1 : (0, 1), S_2 : (1, 0)$ and $S_3 : (1, 1)$. Given the transmission probabilities $p_1, p_2$ and under the fully loaded assumption, we can derive the stationary probabilities $\pi_0, \pi_1, \pi_2, \pi_3$ for the states $S_0, S_1, S_2, S_3$, respectively. For fixed $p_1, p_2, q_1$ and $q_2$, the upper bounds for the set of arrival rates is derived by taking the average over the departure rates of all states. The set of arrival rates for the fully loaded case is given as follows:

$$C_{\text{full}}(p_1, p_2, q_1, q_2) = \{(\lambda_1, \lambda_2) : \lambda_1 < (\pi_1 + \pi_3)q_2 \psi_2 p_2 \bar{p}_1 + [1 - (\pi_2 + \pi_3)q_1] \psi_1 p_1 \bar{p}_2, \lambda_2 < (\pi_2 + \pi_3)q_1 \psi_1 p_1 \bar{p}_2 + [1 - (\pi_1 + \pi_3)q_2] \psi_2 p_2 \bar{p}_1\},$$  \hfill (1)

where $\bar{p}_i = 1 - p_i$.

The throughput of this cooperation strategy depends on the choices of $q_1$ and $q_2$. When a user is reluctant to cooperate, $q_i$ can be set to a value close to 1 and, vice versa, when the user is willing to cooperate. Regardless of the choice of $q_1$ and $q_2$, the sum of the stable throughput in (1) is equal to $\lambda_1 + \lambda_2 < \psi_1 p_1 (1 - p_2) + \psi_2 p_2 (1 - p_1)$, which is the same as that achieved in the non-cooperative system where the fully loaded region is given by $\{(\lambda_1, \lambda_2) : \lambda_1 < \psi_1 p_1 (1 - p_2), \lambda_2 < \psi_2 p_2 (1 - p_1)\}$ [11]. This is expected since only one message can be transmitted successfully during each time slot under the collision model. However, in the cooperative system, we can take the union over all possible values of $q_1$ and $q_2$ to obtain a larger region than the non-cooperative case. In Fig. 2, we show the stability region of the fully loaded system with $p_1 = p_2 = 0.5$ for the channel defined by $\psi_1 = 0.1$ and $\psi_2 = 0.5$. The region bounded by the solid curve is obtained by taking the union over all $q_1, q_2$. The cases with source or relay priority achieve only a point on the line $\lambda_1 + \lambda_2 = \psi_1 p_1 (1 - p_2) + \psi_2 p_2 (1 - p_1)$. Under the fully loaded assumption, the cooperative system with source priority is identical to the system with no cooperation (however, the two systems are not identical without the fully loaded assumption). When priority is given to the source, the throughput of each user is proportional to the quality of its channel while the relay priority approach balances the throughput of both users.

In Fig. 3, we take the union of the stability region of the fully loaded systems for all possible values of $p_1, p_2$. Even with the fully loaded assumption, we show that the cooperative system achieves a much larger region than the non-cooperative system. The case with relay priority is also advantageous when the channel qualities of the two users are not identical, i.e., $\psi_1 \neq \psi_2$, since it increases the throughput of the user with the weaker channel. The region for source priority overlaps with the non-cooperative region since they are identical under the fully loaded assumption. Note that, in the cooperative case, the true stability region is larger than the fully loaded region for certain scenarios. Detailed discussions can be found in [11].

IV. Stability Region of Cooperative Slotted ALOHA without Relay Buffers

In strategy II, we consider the case where each user has only one buffer and the packet received from the source user is always stored at the front of the queue at the cooperative user. Once the packet is successfully passed on to either the base station or the cooperative user, the packet is then dropped.
Adaptive $q_1$ and $q_2$
Source–Priority, $q_1=q_2=1$
Relay–Priority, $q_1=q_2=0$
No cooperation

Fig. 3. The stability region of the two-user cooperative and non-cooperative slotted ALOHA system for $\psi_1 = 0.1$ and $\psi_2 = 0.5$.

from the source user’s buffer, effectively increasing each user’s departure rate. However, each user will also contribute to the arrival of packets at the other user and may potentially cause the other user to become unstable.

Let $\lambda_{ij}$ be the packet arrival rate at user $j$ due to the transmissions by user $i$. For fixed $p_1$ and $p_2$, it is easy to see that at saturation, where both users are fully loaded, the system can achieve the departure rates

$$\mu_{1,\text{full}} = p_1(1 - p_2), \quad \mu_{2,\text{full}} = p_2(1 - p_1)$$

and the arrival rates from the inter-user channel are

$$\lambda_{21,\text{full}} = p_2(1 - \psi_2)(1 - p_1), \quad \lambda_{12,\text{full}} = p_1(1 - \psi_1)(1 - p_2).$$

By Loynes’ Theorem, the system is guaranteed to be stable for $\lambda_1, \lambda_2$ that satisfies the following inequalities:

$$\lambda_1 + \lambda_{21,\text{full}} < \mu_{1,\text{full}}, \quad \lambda_2 + \lambda_{12,\text{full}} < \mu_{2,\text{full}}. \quad (2)$$

Following the approach given in [8], we start from the saturation point and gradually decrease the arrival rate of user 2, while allowing user 1 to remain fully loaded. In this case, the departure rate of user 2 is still equal to $\mu_{2,\text{full}}$. Notice that, as $\lambda_2$ decreases, user 1 has a better opportunity to send a packet to user 2 and, thus, increases the inter-user arrival rate $\lambda_{12}$. However, the increase of $\lambda_{12}$ would not be fast enough to make the system unstable since, if unstable, $\lambda_{12}$ would be equal to $\lambda_{12,\text{full}}$ and that $\lambda_2 + \lambda_{12,\text{full}}$ would be greater or equal to $\mu_2$, which contradicts with (2).

Specifically, we have $\lambda_{12}$ as follows:

$$\lambda_{12} = p_1(1 - \psi_1)(1 - p_2) \frac{\lambda_2 + \lambda_{12}}{\mu_{2,\text{full}}}$$

$$\Rightarrow \lambda_{12} = \frac{p_1(1 - \psi_1)(1 - p_2) - \lambda_2}{1 - p_1 \psi_1}$$

where, from Little’s theorem [5], $\frac{\lambda_2 + \lambda_{12}}{\mu_{2,\text{full}}}$ is the probability that $B_2$ is non-empty. Hence, when user 1 is fully loaded and user 2 is not, the system is stable for $\lambda_1, \lambda_2$ that satisfies

$$\lambda_2 < \mu_{2,\text{full}} - \lambda_{12,\text{full}}$$

$$= p_2(1 - p_1) - p_1(1 - \psi_1)(1 - p_2) \quad (3)$$

$$\lambda_1 < \mu_1 - \lambda_{21}$$

$$= p_1 \left(1 - p_2 \frac{\lambda_2 + \lambda_{12}}{\mu_{2,\text{full}}} \right) - (1 - p_1)(1 - \psi_2)p_2 \frac{\lambda_2 + \lambda_{12}}{\mu_{2,\text{full}}}$$

$$= p_2(1 - p_2)(\psi_1 + \psi_2 - \psi_1 \psi_2) - \frac{1 - (1 - p_2)\psi_1}{1 - p_1 \psi_1} \lambda_1. \quad (4)$$

The set of arrival rates $(\lambda_1, \lambda_2)$ that satisfy these conditions are denoted by $C_{1,\text{full}}(p_1, p_2)$. Similarly, we can start from the saturation point and decrease the arrival rate of user 1. In this case, we obtain the region $C_{2,\text{full}}(p_1, p_2)$, which is given by

$$\lambda_1 < \mu_{1,\text{full}} - \lambda_{21,\text{full}} = p_1(1 - p_2) - p_2(1 - \psi_2)(1 - p_1) \quad (5)$$

$$\lambda_2 < \frac{p_1(1 - p_1)(\psi_1 + \psi_2 - \psi_1 \psi_2)}{1 - p_2 \psi_2} - \frac{1 - (1 - p_2)\psi_1}{1 - p_1 \psi_1} \lambda_2. \quad (6)$$

The stability region is obtained by taking the union of the regions $C_{1,\text{full}}(p_1, p_2)$ and $C_{2,\text{full}}(p_1, p_2)$ over all $p_1, p_2 \in [0, 1]$ [8], i.e., $C = \bigcup_{p_1, p_2} C_{1,\text{full}}(p_1, p_2) \cup C_{2,\text{full}}(p_1, p_2)$.

### Lemma 1

Suppose, for fixed $p_1, p_2$, we have $C_{1,\text{full}}(p_1, p_2)$ $C_{2,\text{full}}(p_1, p_2)$ takes on the form

$$C_{1,\text{full}}(p_1, p_2) = \{(\lambda_1, \lambda_2): \lambda_2 < K_2(p_1, p_2), \lambda_1 < A(p_1) - B(p_1)\lambda_2\}$$

$$C_{2,\text{full}}(p_1, p_2) = \{(\lambda_1, \lambda_2): \lambda_1 < K_1(p_1, p_2), \lambda_2 < C(p_2) - D(p_2)\lambda_1\}$$

The union of these regions over all $p_1, p_2$ is equal to

$$C = \bigcup_{p_1, p_2} \{(\lambda_1, \lambda_2): \lambda_1 < K_1(p_1, p_2), \lambda_2 < K_2(p_1, p_2)\}$$

if the two lines $\lambda_1 = A(p_1) - B(p_1)\lambda_2$ and $\lambda_2 = C(p_2) - D(p_2)\lambda_1$ intersect at the point $(\lambda_1, \lambda_2) = (K_1(p_1, p_2), K_2(p_1, p_2))$ and that

$$C(p_2) = \frac{A(p_1)}{B(p_1)}, \quad D(p_2) = \frac{1}{B(p_1)} \quad (7)$$

for $p_1 + p_2 = 1$.

The proof is omitted but can be found in [12]. It is easy to show that the regions given in (3)-(6) satisfy the conditions of Lemma 1. Therefore, the stability region is characterized by

$$C = \bigcup_{p_1, p_2} \{(\lambda_1, \lambda_2): \lambda_1 = p_1(1 - p_2) - p_2(1 - p_1)(1 - \psi_2),$$

$$\lambda_2 = p_2(1 - p_1) - p_1(1 - p_2)(1 - \psi_1)\} \quad (8)$$

where the boundary is given by the following equation:

$$\lambda_2 = \left(\frac{1 - \sqrt{1 - \psi_2(1 - \lambda_1)}}{\psi_2}\right)^2 \left[1 - (1 - \psi_1)(1 - \psi_2) - (1 - \psi_1)\lambda_1\right].$$

### V. Performance Comparisons and Discussions

In this section, we compare the stability region of the following cases: (i) the cooperative system with relay buffers (for adaptive $q_1, q_2$, source priority, and relay priority); (ii) the cooperative system without relay buffers; and (iii) the non-cooperative slotted ALOHA. In the last case, the boundary of the stability region is given by $\sqrt{\lambda_1/\psi_1} + \sqrt{\lambda_2/\psi_2} = 1$ [11].

In Fig. 4, we set the probability of correct reception as $\psi_1 = \psi_2 = 0.5$. In this setting, the channel of both users are equally bad and, therefore, the advantage of cooperation is less evident. In the case with relay buffers, the fully loaded region for both the source priority approach and the relay
priority approach overlaps with the stability region of the non-cooperative system. This is counter-intuitive for the case with relay priority. Notice that, in this case, at least one user in the system will have an empty relay buffer since, if one is empty and the other is not (i.e., the states $S_1$ and $S_2$), the user with the non-empty relay buffer will always transmit the relay packet and would not have the opportunity to send its own packet to the other user’s relay buffer, until the relay packet is successfully transmitted. For example, in state $S_1$, both users are transmitting the packet originating from user 1. The probability of a successful transmission is the same for either states but the stationary distribution of these states are proportional to the transmission probabilities $p_1$ and $p_2$. Eventually, one can show that the departure rate has the same dependence on the transmission probabilities as in the non-cooperative system. This is not the case when $\psi_1 \neq \psi_2$. In Fig. 5, we set the probability of correct reception as $\psi_1 = 0.1$, $\psi_2 = 0.9$. In this setting, the cooperative strategy with priority given to the relay is no longer identical to the non-cooperative system and, in fact, achieves a much larger region. Please note that, for the case with relay buffers, the region we show is only an inner bound to the true stability region. Yet, it already shows a great improvement over the non-cooperative case.

In Fig. 4, we also plot the case without relay buffers, which is shown to perform worse than the non-cooperative case when $\psi_1 = \psi_2$. In the non-cooperative system, the user with a higher transmission probability will be able to sustain a higher arrival rate. However, without the relay buffer, the packets to be relayed are stored directly at the front of the queue, treating it as a packet of its own. In fact, the user with a higher transmission probability will have a large fraction of packets being referred to the cooperative user and transmitted with the lower transmission probability. This is the reason for the loss in the departure rate on both sides of the line $\lambda_1 = \lambda_2$. However, this approach is advantageous for the user that experiences, on the average, a bad transmission channel. In Fig. 5 where $\psi_1 \ll \psi_2$, the stable throughput of user 1 is increased dramatically, exceeding that of all other cases.

VI. CONCLUSION

The cooperative slotted ALOHA system is proposed with two queuing strategies. In the first scheme, each user is equipped with an additional buffer to store the packets from the other user. In the second scheme, we assume that each user has only one source buffer and the relay packet is stored at the front of the queue upon arrival. The first scheme seems more intelligent and achieves a larger stability region for most cases while the second scheme is simple and requires little change to the original slotted ALOHA system. The analysis on the cooperative system with more than two users will be studied in our future work while considering inter-user channel errors.

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